

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

- 1           1. (Currently amended) A method for finding zeros of a function,  $f$ , within  
2   an interval,  $X$ , using the interval version of Newton's method, wherein  $f'$  is the  
3   derivative of the function  $f$ , the method comprising:  
4           receiving a representation of the interval  $X$ , the representation including a  
5   first floating-point number,  $a$ , representing the left endpoint of  $X$ , and a second  
6   floating-point number,  $b$ , representing the right endpoint of  $X$ ;  
7           storing the representation in a memory of the computer system;  
8           performing an interval Newton step on  $X$ , wherein the point of expansion  
9   is the midpoint,  $x$ , of the interval  $X$ , and wherein performing the interval Newton  
10   step involves evaluating  $f(x)$  to produce an interval result  $f'(x)$ ; and  
11          if  $f'(x)$  contains zero,  
12                  evaluating  $f(a)$  to produce an interval result  $f'(a)$ ,  
13                  evaluating  $f(b)$  to produce an interval result  $f'(b)$ ,  
14                  evaluating a termination condition for the processing of the  
15          current interval  $X$ , wherein the termination condition is TRUE if a  
16          number of conditions are satisfied, including if  $f'(a)$  contains zero  
17          and if  $f'(b)$  contains zero, and  
18                  if the termination condition is TRUE, terminating the  
19          processing of the current interval  $X$ , and recording  $X$  in the memory  
20          as a final bound.

1           2. (Original) The method of claim 1, wherein if  $f'(a)$  does not contain  
2 zero, evaluating  $f(a)$  additionally involves performing an interval Newton step  
3 wherein the point of expansion is  $a$ .

1           3. (Original) The method of claim 1, wherein if  $f'(b)$  does not contain  
2 zero, evaluating  $f(b)$  additionally involves performing an interval Newton step  
3 wherein the point of expansion is  $b$ .

1           4. (Original) The method of claim 1, wherein if  $f'(x)$  contains zero and  
2  $f'(X)$  contains zero, the termination condition for processing the current interval  $X$   
3 is TRUE if  $f'(a)$  contains zero,  $f'(b)$  contains zero,  $f'(x_1)$  contains zero and  $f'(x_2)$   
4 contains zero;  
5           wherein  $x_1$  is the midpoint between  $a$  and  $x$ ; and  
6           wherein  $x_2$  is the midpoint between  $x$  and  $b$ .

1           5. (Original) The method of claim 4, wherein if  $f'(x)$  contains zero and if  
2  $f'(X)$  contains zero, and if either  $f'(x_1)$  or  $f'(x_2)$  does not contain zero, the method  
3 further comprises:  
4           splitting the interval  $X$  in half; and  
5           applying the interval Newton method to each half separately.

1           6. (Original) The method of claim 1, wherein if  $f'(x)$  does not contain zero  
2 and if  $f'(X)$  contains zero, the method further comprises terminating the  
3 processing of the current interval  $X$ , and recording  $X$  as a final bound on condition  
4 that:  
5           the width of the interval  $X$  divided by the magnitude of the interval  $X$  is  
6 less than a first threshold value; and  
7           the magnitude of  $f(X)$  is less than a second threshold value.

1           7. (Original) The method of claim 1, wherein if a given Newton step does  
2 not reduce the width of an interval by at least half, the method further comprises  
3 splitting the interval in half and applying the interval Newton method to each of  
4 the two halves separately.

1           8. (Original) The method if claim 1, wherein if an interval Newton step  
2 results in two intervals, the method further comprises applying the interval  
3 Newton method to each of the two intervals separately.

1           9. (Original) The method of claim 1, wherein if the result of an interval  
2 Newton step is the empty interval, the method returns to process another interval.

1           10. (Original) The method of claim 1, wherein if  $f'(a)$  contains zero and if  
2  $f'(b)$  contains zero, the method returns to process another interval.

1           11. (Original) The method of claim 1,  
2 wherein if  $f'(a)$  does not contain zero or if  $f'(b)$  does not contain zero, the  
3 method further comprises performing an interval Newton step wherein the point  
4 of expansion is the midpoint of the interval  $X$ ; and  
5 wherein if the result of the interval Newton step about the midpoint is the  
6 empty interval, the method returns to process another interval.

1           12. (Original) The method of claim 11, further comprising terminating the  
2 processing of the current interval  $X$  after a predetermined number of iterations.

1           13. (Original) A computer-readable storage medium storing instructions  
2 that when executed by a computer cause the computer to perform a method for  
3 finding zeros of a function,  $f$ , within an interval,  $X$ , using the interval version of

4 Newton's method, wherein  $f'$  is the derivative of the function  $f$ , the method  
5 comprising:  
6 receiving a representation of the interval  $X$ , the representation including a  
7 first floating-point number,  $a$ , representing the left endpoint of  $X$ , and a second  
8 floating-point number,  $b$ , representing the right endpoint of  $X$ ;  
9 performing an interval Newton step on  $X$ , wherein the point of expansion  
10 is the midpoint,  $x$ , of the interval  $X$ , and wherein performing the interval Newton  
11 step involves evaluating  $f(x)$  to produce an interval result  $f'(x)$ ; and  
12 if  $f'(x)$  contains zero,  
13 evaluating  $f(a)$  to produce an interval result  $f'(a)$ ,  
14 evaluating  $f(b)$  to produce an interval result  $f'(b)$ ,  
15 evaluating a termination condition for the processing of the  
16 current interval  $X$ , wherein the termination condition is TRUE if a  
17 number of conditions are satisfied, including if  $f'(a)$  contains zero  
18 and if  $f'(b)$  contains zero, and  
19 if the termination condition is TRUE, terminating the  
20 processing of the current interval  $X$ , and recording  $X$  as a final  
21 bound.

1 14. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f'(a)$  does not contain zero evaluating  $f(a)$  additionally involves  
3 performing an interval Newton step wherein the point of expansion is  $a$ .

1 15. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f'(b)$  does not contain zero, evaluating  $f(b)$  additionally involves  
3 performing an interval Newton step wherein the point of expansion is  $b$ .

1           16. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f'(x)$  contains zero and  $f'(X)$  contains zero, the termination condition  
3 for processing the current interval  $X$  is TRUE if  $f'(a)$  contains zero,  $f'(b)$  contains  
4 zero,  $f'(x_1)$  contains zero and  $f'(x_2)$  contains zero;  
5           wherein  $x_1$  is the midpoint between  $a$  and  $x$ ; and  
6           wherein  $x_2$  is the midpoint between  $x$  and  $b$ .

1           17. (Original) The computer-readable storage medium of claim 16,  
2 wherein if  $f'(x)$  contains zero and if  $f'(X)$  contains zero, and if either  $f'(x_1)$  or  $f$   
3  $'(x_2)$  does not contain zero, the method further comprises:  
4           splitting the interval  $X$  in half; and  
5           applying the interval Newton method to each half separately.

1           18. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f'(x)$  does not contain zero and if  $f'(X)$  contains zero, the method  
3 further comprises terminating the processing of the current interval  $X$ , and  
4 recording  $X$  as a final bound on condition that:  
5           the width of the interval  $X$  divided by the magnitude of the interval  $X$  is  
6 less than a first threshold value; and  
7           the magnitude of  $f(X)$  is less than a second threshold value.

1           19. (Original) The computer-readable storage medium of claim 13,  
2 wherein if a given Newton step does not reduce the width of an interval by at least  
3 half, the method further comprises splitting the interval in half and applying the  
4 interval Newton method to each of the two halves separately.

1           20. (Original) The computer-readable storage medium of claim 13, wherein  
2 if an interval Newton step results in two intervals, the method further comprises  
3 applying the interval Newton method to each of the two intervals separately.

1           21. (Original) The computer-readable storage medium of claim 13,  
2 wherein if the result of an interval Newton step is the empty interval, the method  
3 returns to process another interval.

1           22. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f^I(a)$  contains zero and if  $f^I(b)$  contains zero, the method returns to  
3 process another interval.

1           23. (Original) The computer-readable storage medium of claim 13,  
2 wherein if  $f^I(a)$  does not contain zero or if  $f^I(b)$  does not contain zero, the  
3 method further comprises performing an interval Newton step wherein the point  
4 of expansion is the midpoint of the interval  $X$ ; and  
5 wherein if the result of the interval Newton step about the midpoint is the  
6 empty interval, the method returns to process another interval.

1           24. (Original) The computer-readable storage medium of claim 13,  
2 wherein the method further comprises terminating the processing of the current  
3 interval  $X$  after a predetermined number of iterations.

1           25. (Currently amended) An apparatus that finds zeros of a function,  $f$ ,  
2 within an interval,  $X$ , using the interval version of Newton's method, wherein  $f'$  is  
3 the derivative of the function  $f$ , the apparatus comprising:  
4 a receiving mechanism that is configured to receive a representation of the  
5 interval  $X$ , the representation including a first floating-point number,  $a$ ,

6 representing the left endpoint of  $X$ , and a second floating-point number,  $b$ ,  
7 representing the right endpoint of  $X$ ;  
8 a storing mechanism configured to store the representation in a memory of  
9 the computer system;  
10 an interval Newton mechanism that is configured to perform an interval  
11 Newton step on  $X$ , wherein the point of expansion is the midpoint,  $x$ , of the  
12 interval  $X$ , and wherein performing the interval Newton step involves evaluating  
13  $f(x)$  to produce an interval result  $f'(x)$ ; and  
14 wherein if  $f'(x)$  contains zero, the interval Newton mechanism is  
15 configured to,  
16 evaluate  $f(a)$  to produce an interval result  $f'(a)$ ,  
17 evaluate  $f(b)$  to produce an interval result  $f'(b)$ ,  
18 evaluate a termination condition for the processing of the  
19 current interval  $X$ , wherein the termination condition is TRUE if a  
20 number of conditions are satisfied, including if  $f'(a)$  contains zero  
21 and if  $f'(b)$  contains zero, and  
22 if the termination condition is TRUE, to terminate the  
23 processing of the current interval  $X$ , and to record  $X$  in the memory  
24 as a final bound.

1 26. (Original) The apparatus of claim 25, wherein while evaluating  $f(a)$  to  
2 produce the interval result  $f'(a)$ , the interval Newton mechanism is additionally  
3 configured to perform an interval Newton step wherein the point of expansion is  $a$   
4 if  $f'(a)$  does not contain zero.

1 27. (Original) The apparatus of claim 25, wherein while evaluating  $f(b)$  to  
2 produce the interval result  $f'(b)$  the interval Newton mechanism is additionally

3 configured to perform an interval Newton step wherein the point of expansion is  $b$   
4 if  $f'(b)$  does not contain zero.

1 28. (Original) The apparatus of claim 25, wherein if  $f'(x)$  contains zero and  
2  $f'(X)$  contains zero, the termination condition for processing the current interval  
3  $X$  is TRUE if  $f'(a)$  contains zero,  $f'(b)$  contains zero,  $f'(x_1)$  contains zero and  
4  $f'(x_2)$  contains zero;  
5 wherein  $x_1$  is the midpoint between  $a$  and  $x$ ; and  
6 wherein  $x_2$  is the midpoint between  $x$  and  $b$ .

1 29. (Original) The apparatus of claim 28, wherein if  $f'(x)$  contains zero and  
2 if  $f'(X)$  contains zero, and if either  $f'(x_1)$  or  $f'(x_2)$  does not contain zero, the  
3 interval Newton mechanism is additionally configured to:  
4 split the interval  $X$  in half; and to  
5 apply the interval Newton method to each half separately.

1 30. (Original) The apparatus of claim 25, wherein if  $f'(x)$  does not contain  
2 zero and if  $f'(X)$  contains zero, the interval Newton mechanism is additionally  
3 configured to terminate the processing of the current interval  $X$ , and to record  $X$  as  
4 a final bound on condition that:  
5 the width of the interval  $X$  divided by the magnitude of the interval  $X$  is  
6 less than a first threshold value; and  
7 the magnitude of  $f(X)$  is less than a second threshold value.

1 31. (Original) The apparatus of claim 25, wherein if a given Newton step  
2 does not reduce the width of an interval by at least half, the interval Newton  
3 mechanism is additionally configured to split the interval in half, and to apply the  
4 interval Newton method to each of the two halves separately.



1           32. (Original) The apparatus of claim 25, wherein if an interval Newton  
2 step results in two intervals, the interval Newton mechanism is additionally  
3 configured to apply the interval Newton method to each of the two intervals  
4 separately.

1           33. (Original) The apparatus of claim 25, wherein if the result of an  
2 interval Newton step is the empty interval, the interval Newton mechanism is  
3 additionally configured to return to process another interval.

1           34. (Original) The apparatus of claim 25, wherein if  $f'(a)$  contains zero  
2 and if  $f'(b)$  contains zero, the interval Newton mechanism is additionally  
3 configured to return to process another interval.

1           35. (Original) The apparatus of claim 25,  
2 wherein if  $f'(a)$  does not contain zero or if  $f'(b)$  does not contain zero, the  
3 interval Newton mechanism is additionally configured to perform an interval  
4 Newton step wherein the point of expansion is the midpoint of the interval  $X$ ; and  
5 wherein if the result of the interval Newton step about the midpoint is the  
6 empty interval, the interval Newton mechanism is additionally configured to  
7 return to process another interval.

1           36. (Original) The apparatus of claim 35, wherein the interval Newton  
2 mechanism is additionally configured to terminate the processing of the current  
3 interval  $X$  after a predetermined number of iterations.